Analysis of Indeterminate Contact Forces in Robotic Grasping and Contact Tasks

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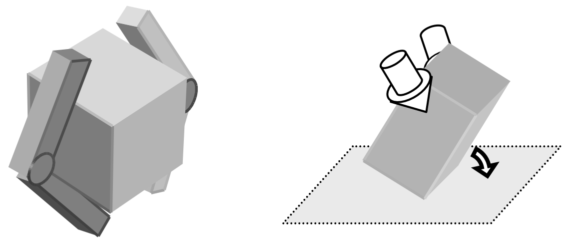
Background

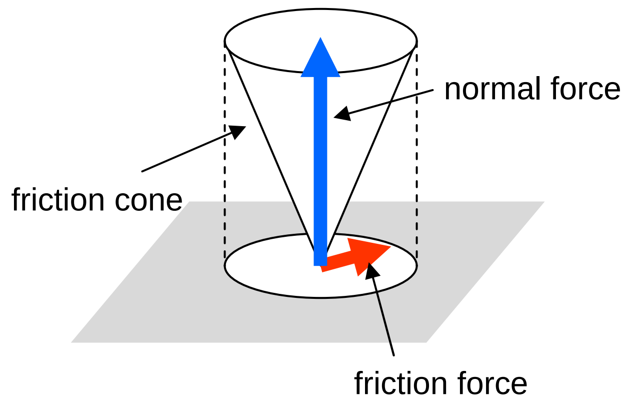
- Analysis of contact forces:
  necessary in various fields in robotics
  - Grasping
  - Manipulation
  - Fixturing
  - Walking



- Rigid-body + Coulomb friction
  - simple and widely used
  - Contact forces can be indeterminate

Coulomb friction

- A friction force must be within its friction cone
  - "Local" constraint on the feasibility

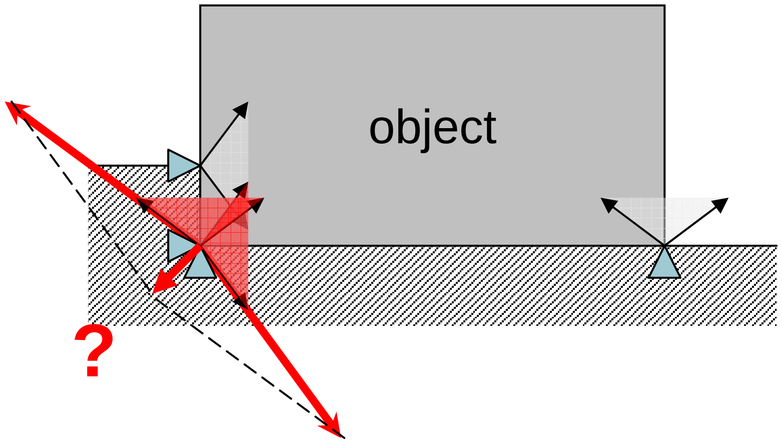


- Question: Is **every combination** of locally-feasible friction forces also feasible?

Answer: No!

Combination of friction forces

(Example)



⇨ Combinations of locally-feasible contact forces may be infeasible

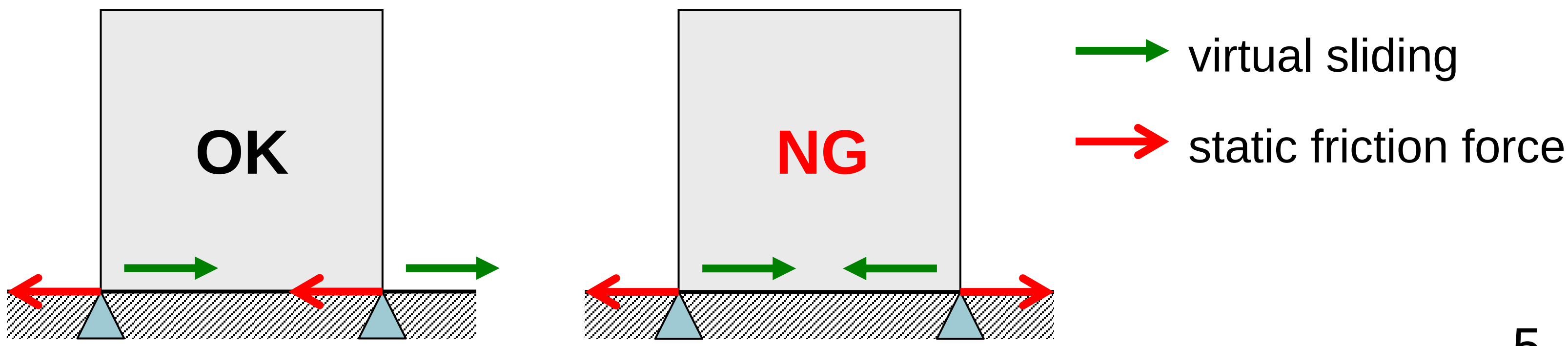
("Global" constraint on **combinations** of friction forces)

Constraint on combinations of friction forces

[Omata and Nagata 00 T-RA]
[Omata 01 ICRA]

- A static friction force acts only in the opposite direction of the trend of virtual sliding
- Only a part of combinations of virtual slidings are feasible due to contact kinematics

⇨ **Only a part of combinations of static friction forces are feasible**



Relationship between sliding and friction

- **Virtual** sliding must be distinguished from **actual** sliding

Contacts in actual sliding	Contacts not in actual sliding	
	in virtual sliding	not in virtual sliding
Kinetic friction	Static friction	No friction
	(Actual sliding is prevented by static friction)	(No need to prevent sliding by static friction)

Omata's formulation [Omata 01 ICRA]

- Constraint on virtual sliding

$$\begin{bmatrix} \boldsymbol{W}^T & \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{V} \\ -\dot{\boldsymbol{\theta}} \end{bmatrix} = \boldsymbol{T}\dot{\boldsymbol{Y}}$$

wrench matrix

virtual object velocity /angular velocity

virtual sliding velocity

Jacobian

virtual joint angular velocity

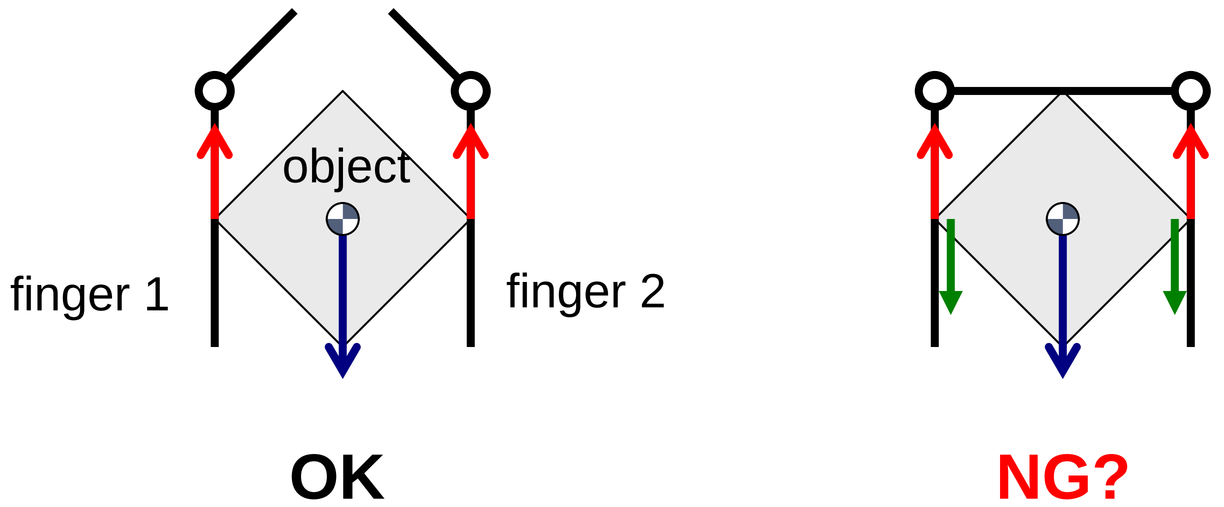
contact tangents

  - Virtual sliding velocity $(\dot{\boldsymbol{Y}})$ that satisfies this constraint is feasible

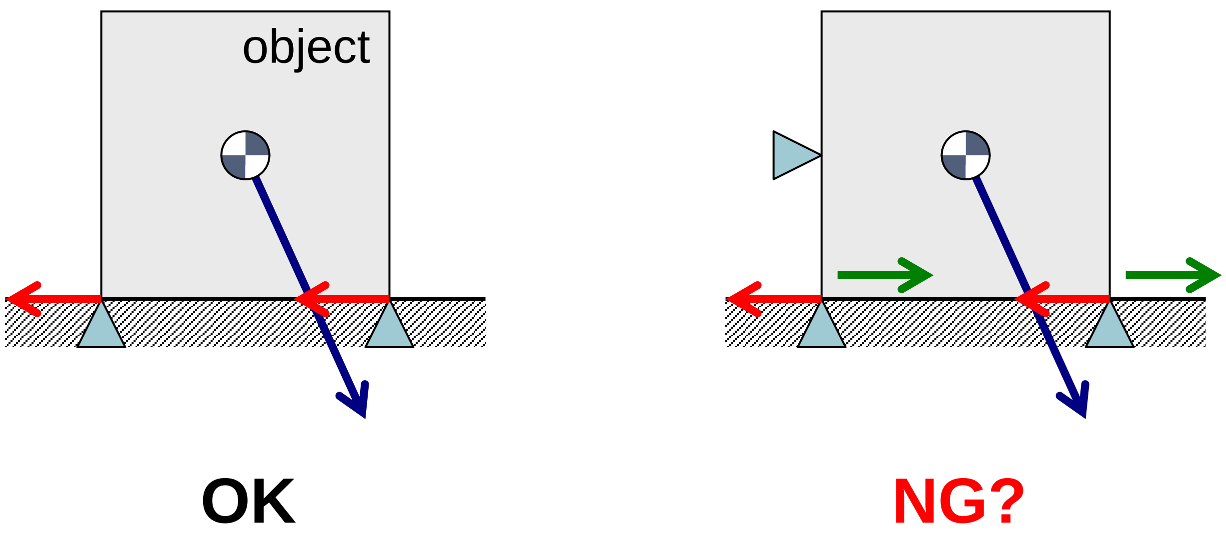
⇨ We can find feasible friction forces by investigating feasible $\dot{\boldsymbol{Y}}$

Unreasonable results in Omata's formulation (1)

- Adding a contact point makes a grasp infeasible



Unreasonable results in Omata's formulation (2)



Objective

- To modify Omata's formulation to exclude such unreasonable results
  - We present a procedure to calculate the set of possible indeterminate contact forces
  - We also present a technique for computation reduction

Why such unreasonable results?

- Problem in Omata's formulation: Constraint on **actual** instantaneous sliding is applied to **virtual** sliding

$$\begin{bmatrix} W^T & J \end{bmatrix} \begin{bmatrix} V \\ -\dot{\theta} \end{bmatrix} = T\dot{Y}$$

  - Some valid virtual slidings are excluded
  - Thus some feasible friction forces are also excluded

⇨ We need a more **relaxed** constraint on virtual sliding

A new relaxed constraint (1)

- Constraint on virtual sliding of each **subset** of contacts

$$\boldsymbol{B}\begin{bmatrix}\boldsymbol{W}^T & \boldsymbol{J}\end{bmatrix}\begin{bmatrix}\boldsymbol{V} \\ -\dot{\boldsymbol{\theta}}\end{bmatrix} = \boldsymbol{T}\dot{\boldsymbol{Y}}$$

$\boldsymbol{B} := \mathrm{diag}(b_1\boldsymbol{I}_3, \ldots, b_M\boldsymbol{I}_3) \in \Re^{3M\times 3M}$: selection matrix

$$b_i = \begin{cases} 1 & \text{when } P_i \text{ is in virtual sliding,} \\ 0 & \text{otherwise.} \end{cases}$$

When $\boldsymbol{B} = \boldsymbol{I}$: Identical solution to Omata's formulation
Otherwise: Additional solutions may be found

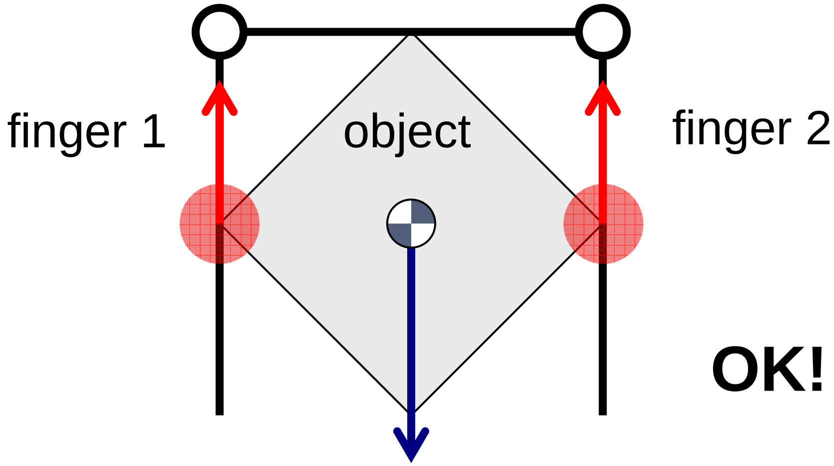
A new relaxed constraint (2)

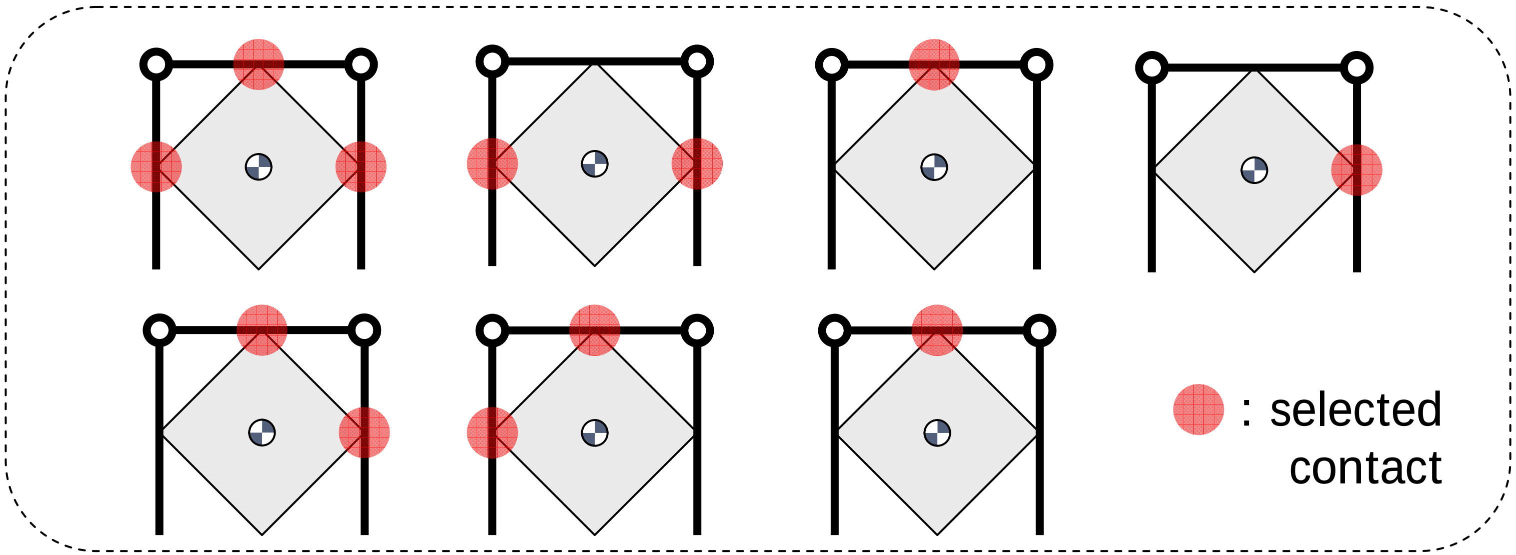
$$\boldsymbol{B}\begin{bmatrix}\boldsymbol{W}^T & \boldsymbol{J}\end{bmatrix}\begin{bmatrix}\boldsymbol{V} \\ -\dot{\boldsymbol{\theta}}\end{bmatrix} = \boldsymbol{T}\dot{\boldsymbol{Y}}$$

$\boldsymbol{B}$: selection matrix

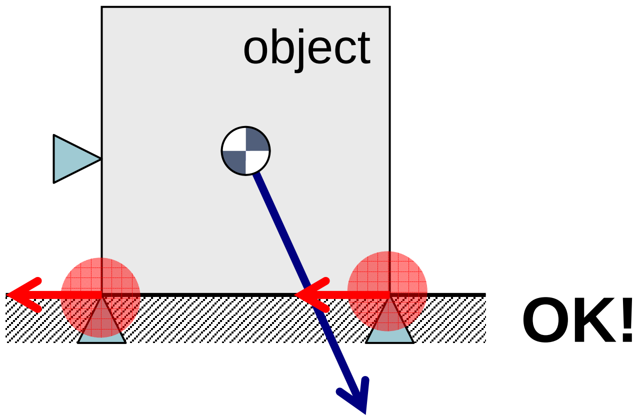
- Only selected contact points are constrained by contact kinematics
- Unselected contact points cannot generate static friction forces
- Consider **every** subset of contacts to obtain total set of contact forces

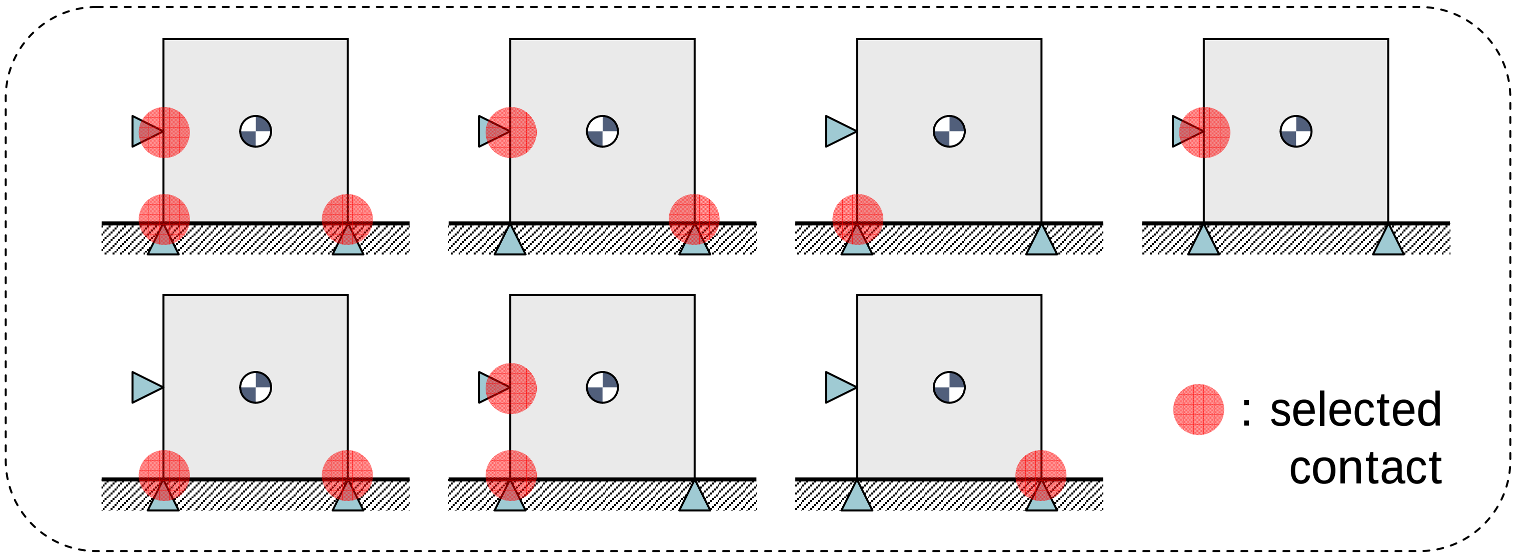
Example: Application of new constraint (1)





Example: Application of new constraint (2)





Procedure to calculate the set of indeterminate contact forces

- Set of possible indeterminate contact forces for each subcase: solution of linear inequalities

$$\begin{cases} \boldsymbol{f} = \boldsymbol{C}\boldsymbol{k}, \quad \boldsymbol{k} \geq \boldsymbol{0} \\ \boldsymbol{A}^T \boldsymbol{f} = \boldsymbol{w} \\ \boldsymbol{T}^T(\boldsymbol{I}_{3M} - \boldsymbol{B})\boldsymbol{f} = \boldsymbol{0} \\ \boldsymbol{S}\boldsymbol{T}^T \boldsymbol{f} \leq \boldsymbol{0} \end{cases}$$

Coulomb friction

equilibrium equation

unselected contacts cannot generate friction forces

friction forces act only to prevent virtual slidings

- Total set of possible indeterminate contact forces: **union** of the above solutions

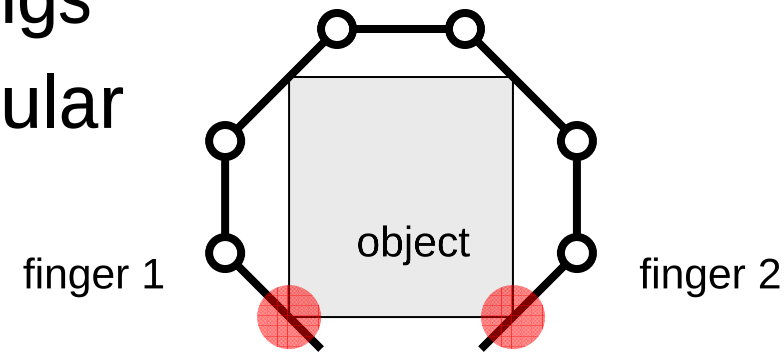
Problem in new formulation

- Calculation for every subset of contact points
  - Combinatorial computation
  - Exponential complexity w.r.t. the number of contact points

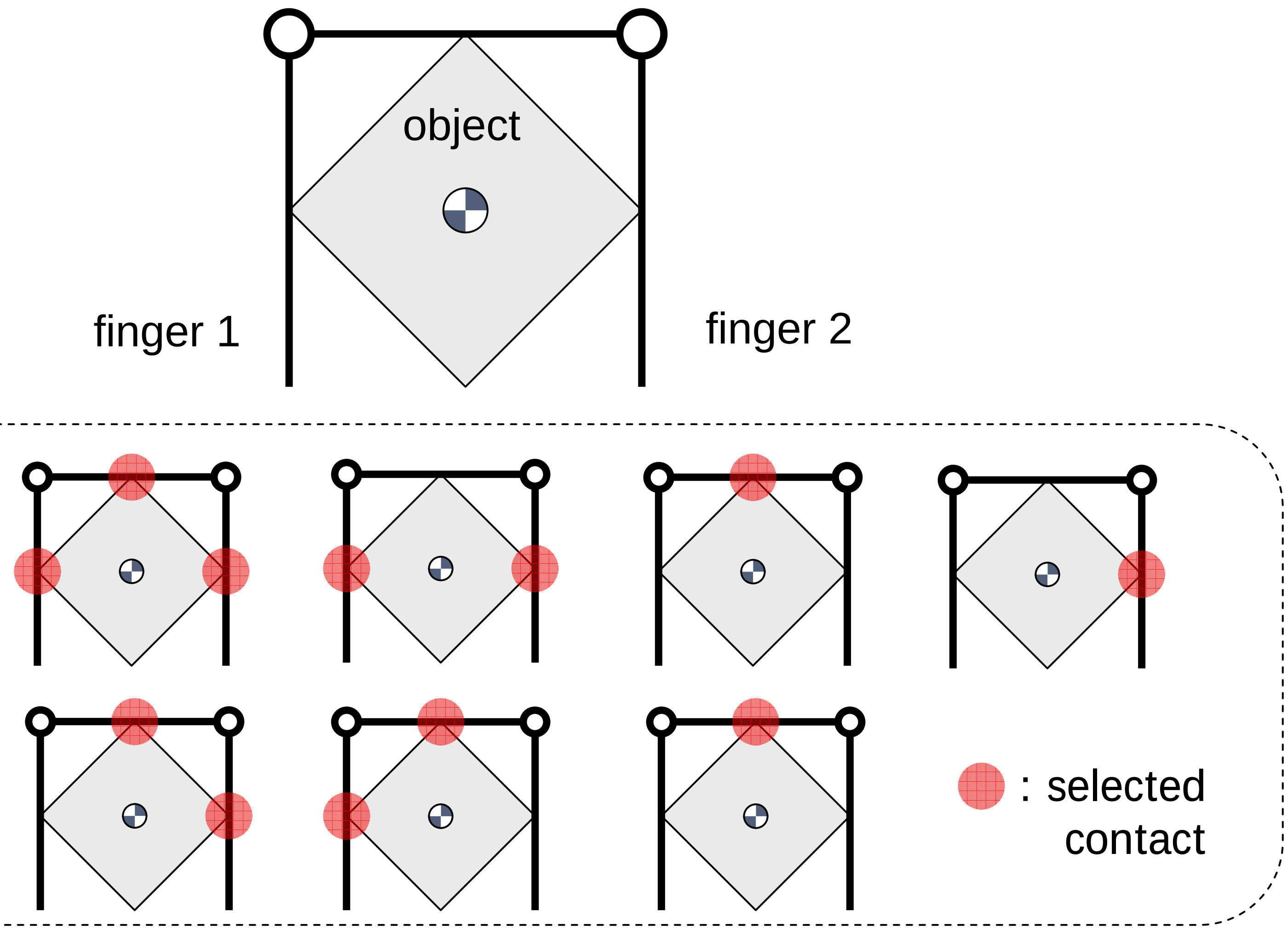
⇨ Computation reduction is desired

A sufficient condition to skip unnecessary subsets

- Fingertip link has only one contact point and a matrix Z is nonsingular
- In this case, we can safely ignore the subsets in which the contact point is unselected
  - Proof: see proceedings
  - Z is usually nonsingular

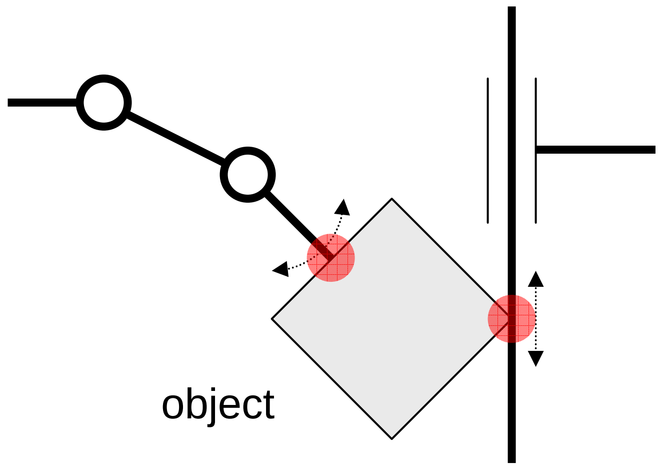


Example: Computation reduction



Special cases where computation reduction is inapplicable

- Cases where Z is singular:
  - Contact point motion caused by outermost joint is in contact tangent space



Summary

- Omata's formulation on indeterminate contact forces generates unreasonable results in some cases
- A modified formulation to exclude such unreasonable results is proposed
  - Procedure to calculate the set of possible indeterminate contact forces
  - Technique for computation reduction

Future work：

• Application to various problems of robotic grasping and manipulation [Maeda 05 ICRA] [Maeda 06 ICRA] [Makita 07 ISHF]

• Further computation reduction