Analysis of Object-Stability and Internal Force in Robotic Contact Tasks

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2. Modeling of System
3. Evaluation of Object-Stability
4. Judging Possibility of Excessive Internal Force
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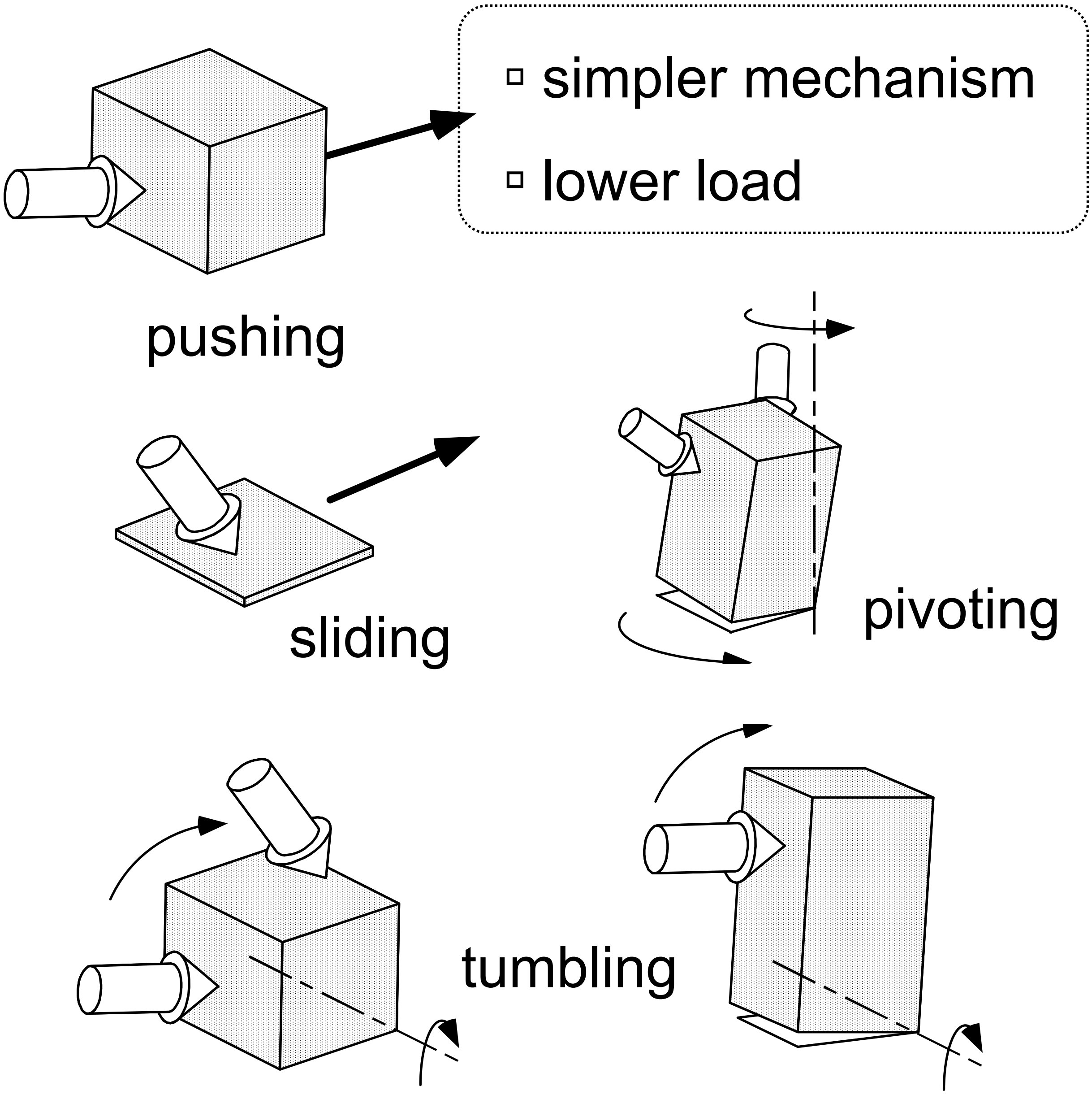
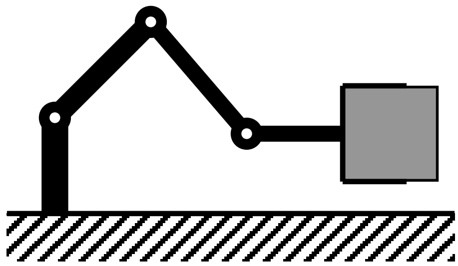
1. Introduction

Graspless Manipulation

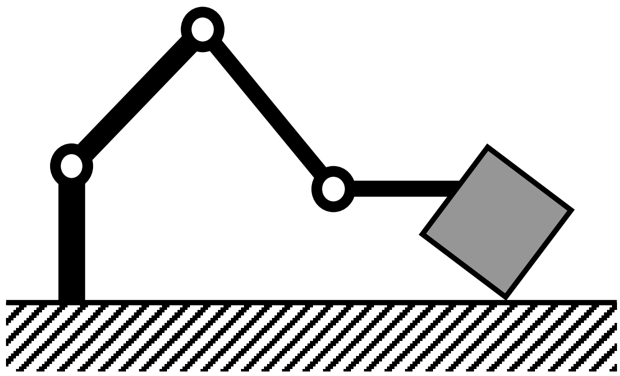


Pick-and-Place:
Once have stably grasped, it's OK

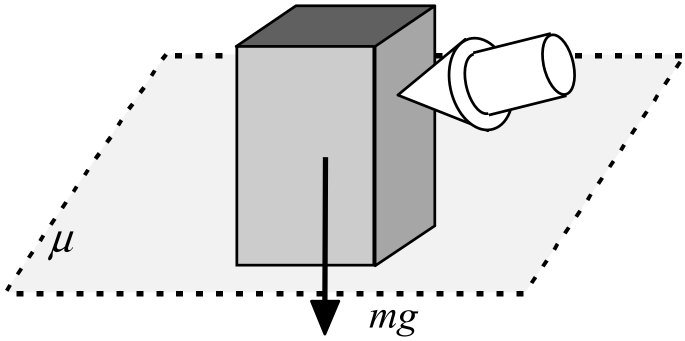


Graspless Manipulation

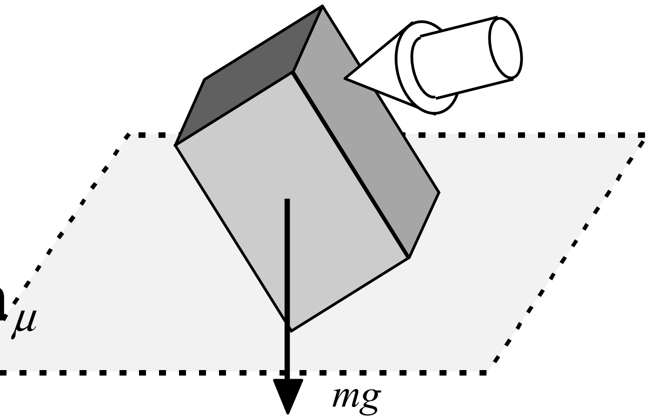
It is always required
to pay attention to the stability



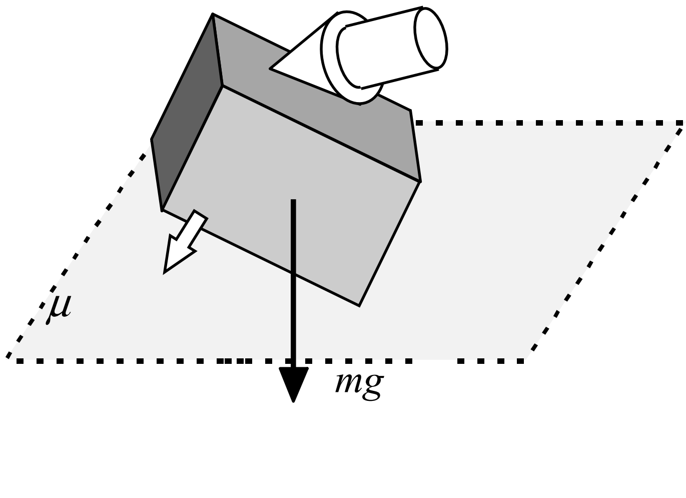
Stable Equilibrium



Unstable Equilibrium



Non-Equilibrium



Stability measure
for graspless manipulation

• needs to take account of gravitational and frictional force

We adopt:

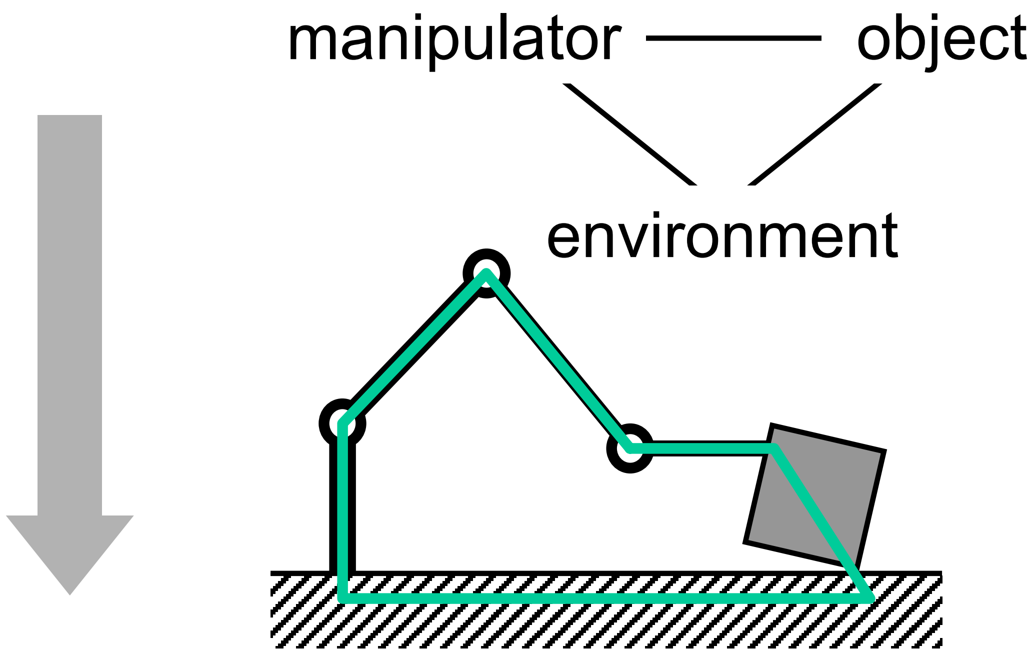
The minimum magnitude of external wrench required to break equilibrium

and propose:

An algorithm to calculate the stability measure by linear programming

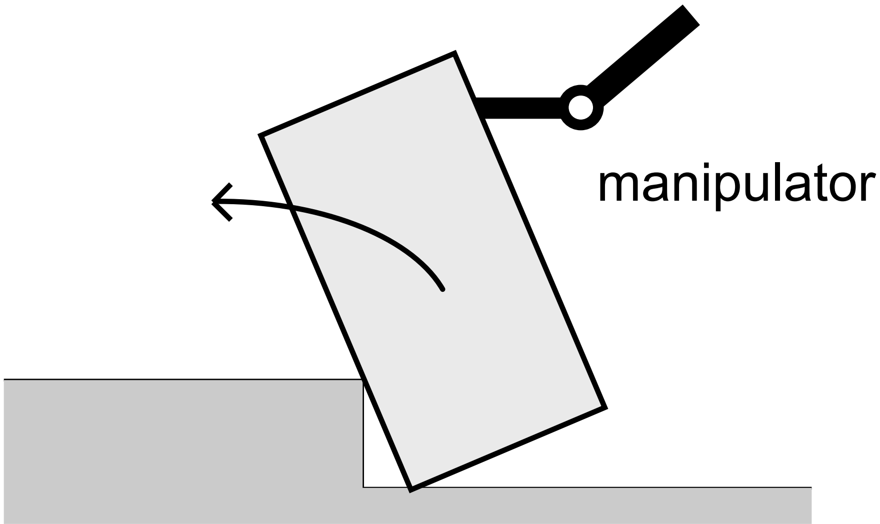
Graspless Manipulation

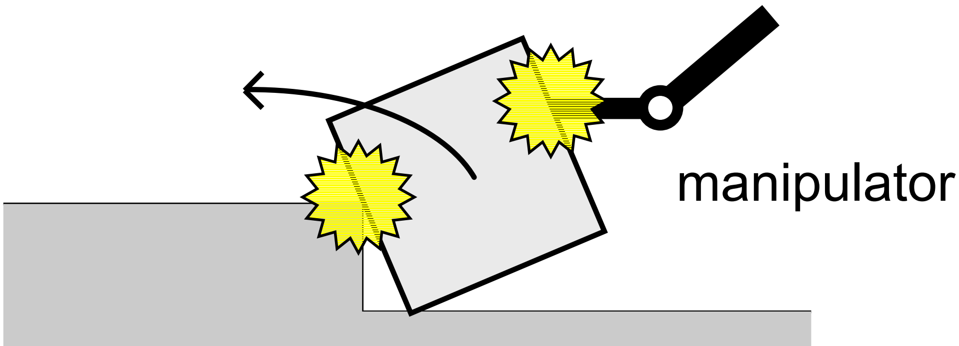
There exists a closed loop



<u>It is necessary to pay attention to the possibility of excessive internal force</u>

(in case of using <u>position-controlled manipulator</u>)





We also propose:

An algorithm to judge the possibility
of excessive internal force
by linear programming

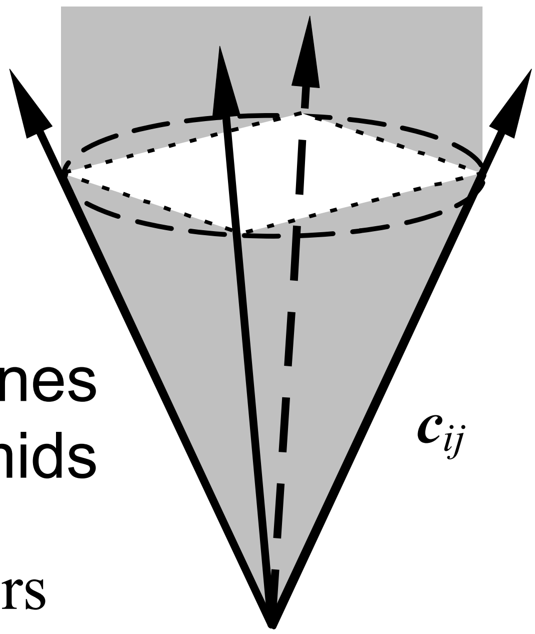
2. Modeling of System

Assumptions

- Object, robots, and environment are rigid
- Coulomb friction exists at the contacts
- Position-controlled robots are regarded as equivalent to environment

Approximate friction cones with pyramids

$c_{i1}, \dots, c_{is}$: unit edge vectors



- force at the contact point with environment

$$\boldsymbol{f}_i = k_{i1}\boldsymbol{c}_{i1} + \cdots + k_{is}\boldsymbol{c}_{is} = \boldsymbol{C}_i\boldsymbol{k}_i \quad (k_{i1}, \cdots, k_{is} \geq 0)$$

- force at the contact point with force-controlled robot

$$\boldsymbol{f}_i = k_{i1}\boldsymbol{c}_{i1} + \cdots + k_{is}\boldsymbol{c}_{is} = \boldsymbol{C}_i\boldsymbol{k}_i \quad (k_{i1}, \cdots, k_{is} \geq 0)$$

subject to $\boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{f}_i = \boldsymbol{J}_i^T \boldsymbol{C}_i \boldsymbol{k}_i$

$\boldsymbol{\tau}_i$: joint torque vector

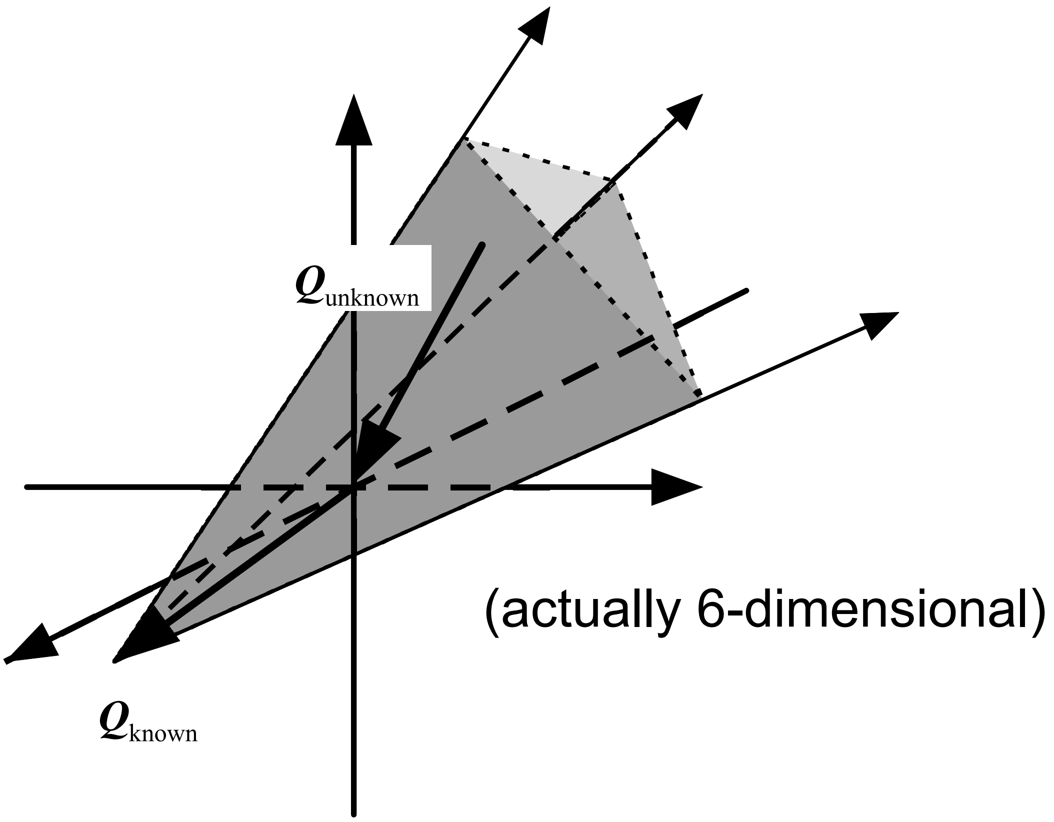
$\boldsymbol{J}_i$: Jacobian matrix

Total wrench applied to the object, $\boldsymbol{Q}$:

$$\boldsymbol{Q} = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k}$$
$$\left(\text{subject to } \boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{C}_i \boldsymbol{k}_i \quad (i = n+1, \ldots, m)\right)$$

Set of possible values of $\boldsymbol{Q}$:

convex polyhedron in wrench space



3. Evaluation of Object-Stability

Our Stability Measure z :

$$z = -\max_{\|\Delta\boldsymbol{q}\|=1} \left\{ \min_{\substack{\boldsymbol{k} \geq \boldsymbol{0} \\ \boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{C}_i \boldsymbol{k}_i \ (i=n+1,\ldots,m)}} (\boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k})^T \Delta\boldsymbol{q} \right\}$$

where $\|\Delta\boldsymbol{q}\| = \sqrt{\Delta\boldsymbol{q}^T \boldsymbol{M} \Delta\boldsymbol{q}} = \left(\|\boldsymbol{K}\Delta\boldsymbol{q}\|_2\right)$

$\boldsymbol{M} = \boldsymbol{K}^T \boldsymbol{K}$: inertia matrix of the object

z means:

Geometrically,

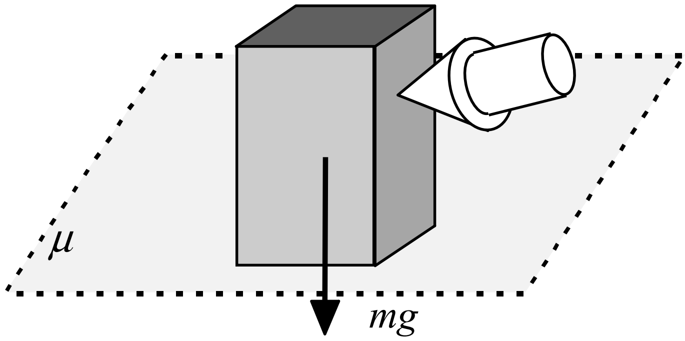
the minimum distance
from the origin of wrench space
to the surface of the polyhedron

Physically,

the minimum magnitude of
external wrench required
to break equilibrium

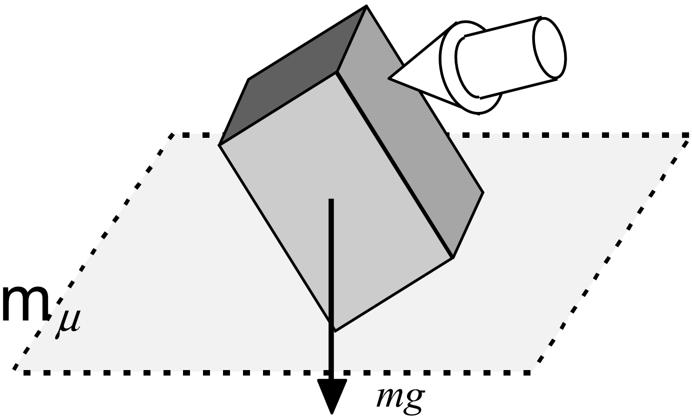
$$z > 0$$

Stable Equilibrium
(Gravity Closure)



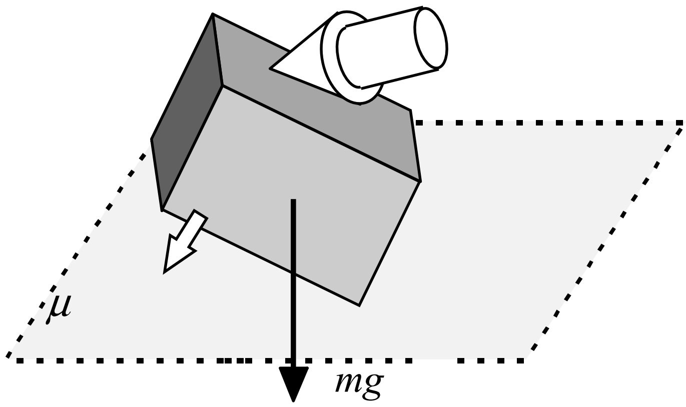
$$z = 0$$

Unstable Equilibrium



$$z < 0$$

Non-Equilibrium



2-norm $\|\Delta\boldsymbol{q}\| = \|\boldsymbol{K}\Delta\boldsymbol{q}\|_2 = \sqrt{\sum \left|(\boldsymbol{K}\Delta\boldsymbol{q})_i\right|^2}$

⇩ approximation

1-norm $\|\Delta\boldsymbol{q}\| = \|\boldsymbol{K}\Delta\boldsymbol{q}\|_1 = \sum \left|(\boldsymbol{K}\Delta\boldsymbol{q})_i\right|$

⇩

z can be calculated by <u>linear programming</u>

$$\text{maximize} \quad z'$$

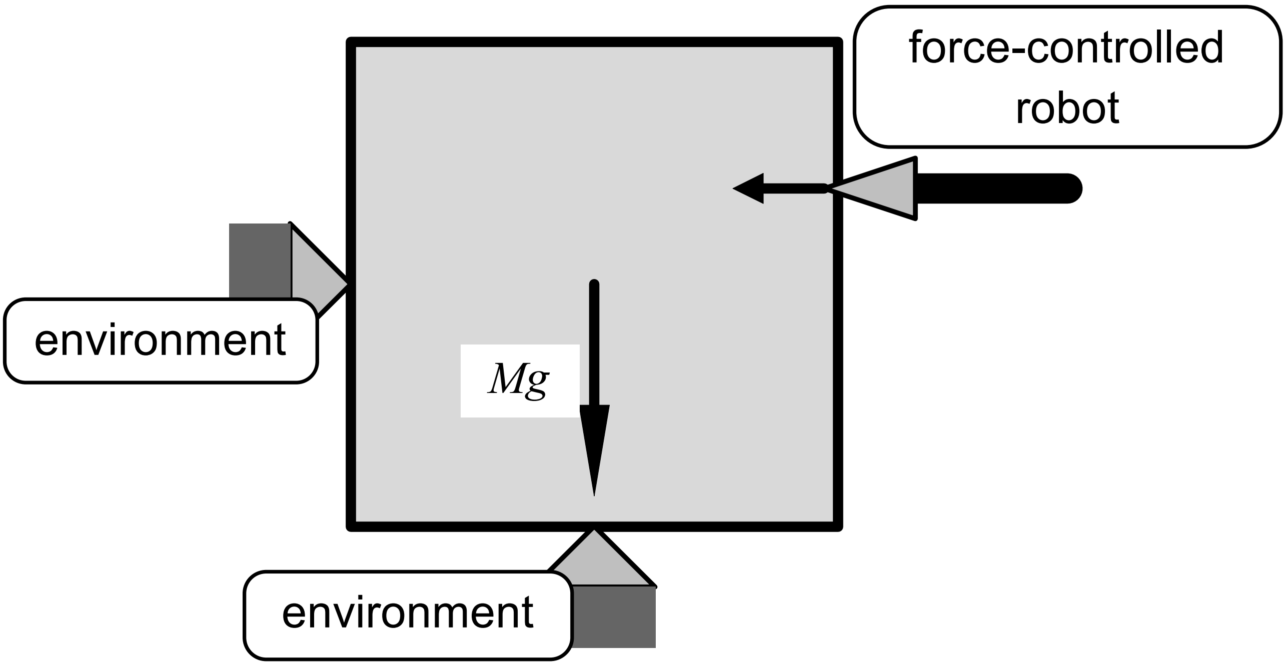
subject to

$$\begin{cases} z'\boldsymbol{l}_1 = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k}'_1, & -z'\boldsymbol{l}_1 = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k}'_7 \\ \quad\vdots & \quad\vdots \\ z'\boldsymbol{l}_6 = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k}'_6, & -z'\boldsymbol{l}_6 = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}\boldsymbol{k}'_{12} \\ \boldsymbol{k}'_1, \ldots, \boldsymbol{k}'_{12} \geq \boldsymbol{0} & (\boldsymbol{k}'_i = [\boldsymbol{k}'^T_{i1} \ \cdots \ \boldsymbol{k}'^T_{im}]^T) \\ \boldsymbol{\tau}_j = \boldsymbol{J}_j^T \boldsymbol{C}_j \boldsymbol{k}'_{ij} & (i = 1, \ldots, 12, \ j = n+1, \ldots, m) \end{cases}$$

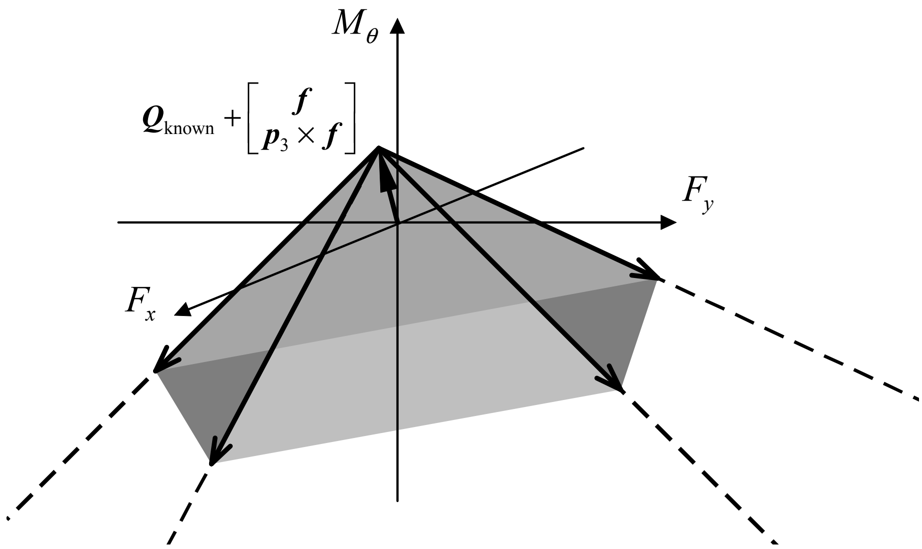
where

$$\boldsymbol{l}_i = \boldsymbol{K}[0 \ \cdots \ 0 \ \underset{\hat{i}}{1} \ 0 \ \cdots \ 0]^T \ (i = 1, \ldots, 6)$$

Numerical Example:
square object in contact task



- friction coefficient = 0.25
- weight of the object = 1 N
- force applied by the robot = 0.3 N



Possible Value of $\boldsymbol{Q}$ (contact forces + gravity)
in Wrench Space

Stability Measure

- 1-norm approximation $z = 0.197$

(the weakest direction) $\Delta \boldsymbol{q} = [0 \quad 0 \quad 1]^T$

- 2-norm case $z = 0.186$

(the weakest direction) $\Delta \boldsymbol{q} = [0.17 \;\; 0.28 \;\; 0.95]^T$

4. Judging Possibility of Excessive Internal Force

Solving the following linear programming problem:

$$\text{maximize } \boldsymbol{c}^T \boldsymbol{k} \qquad (\boldsymbol{c} = [1 \ \cdots \ 1]^T)$$

$$\text{subject to} \begin{cases} \boldsymbol{W}_0 \boldsymbol{k} = \boldsymbol{0} \\ \boldsymbol{k} \geq \boldsymbol{0} \end{cases}$$

$\boldsymbol{W}_0$: formed by excluding some columns from $\boldsymbol{W}$

$\boldsymbol{c}^T \boldsymbol{k} \to \infty$

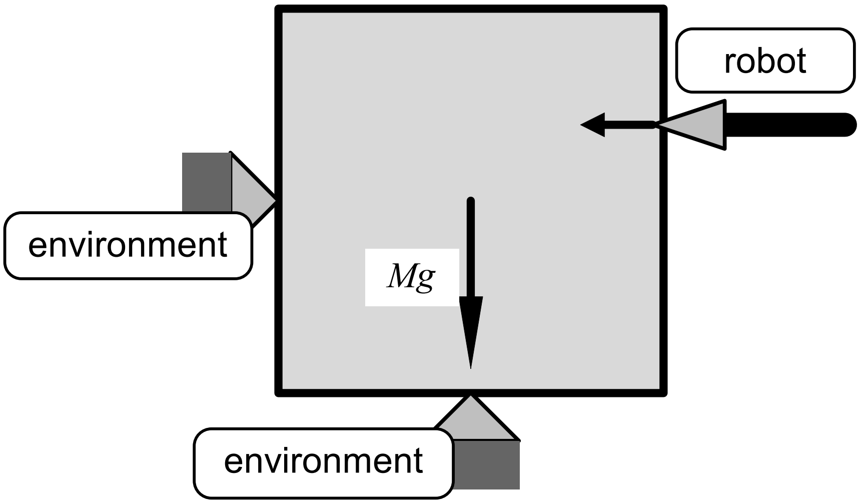
excessive internal force *may* be generated

equilibrium is maintained even if infinite internal force is generated

$\boldsymbol{c}^T \boldsymbol{k} = 0$

excessive internal force is *never* generated

Numerical Example



- if the robot is *force*-controlled

$$W_0 = \frac{1}{\sqrt{17}}\begin{bmatrix} 1 & -1 & 4 & 4 \\ 4 & 4 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

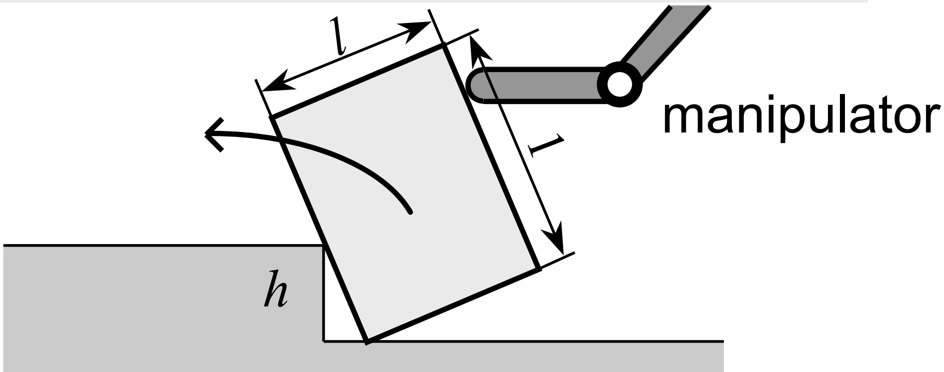
⇨ excessive internal force is *never* generated

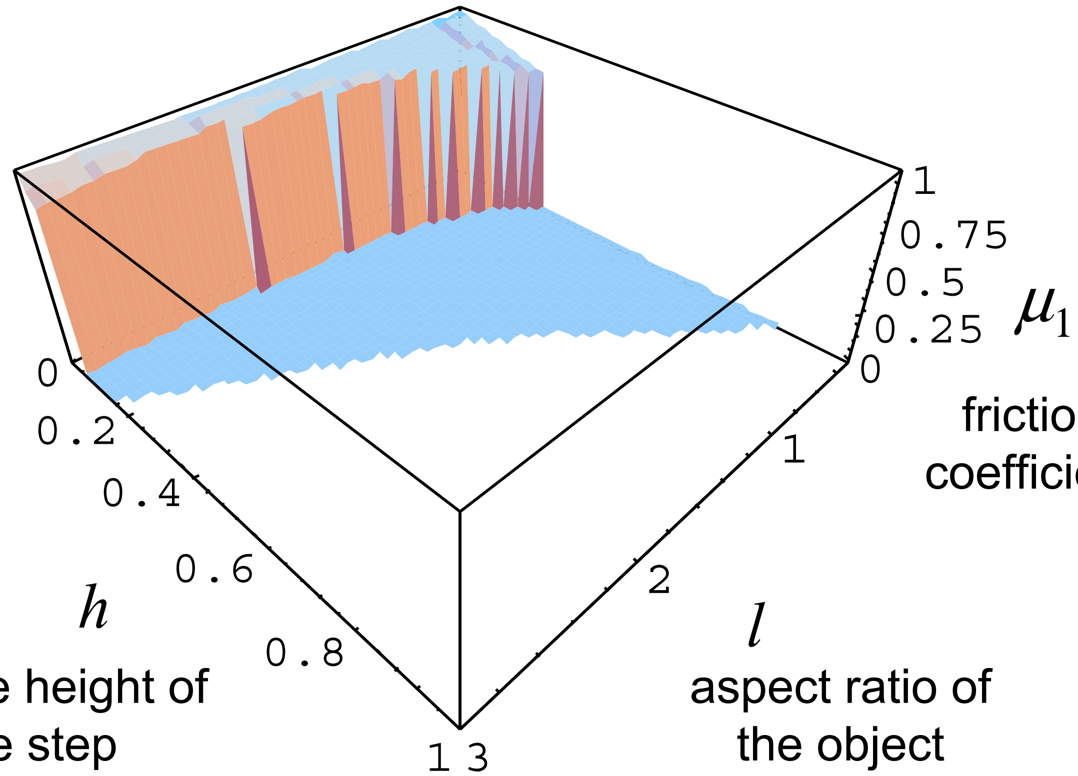
- if the robot is *position*-controlled

$$W_0 = \frac{1}{\sqrt{17}}\begin{bmatrix} 1 & -1 & 4 & 4 & -4 & -4 \\ 4 & 4 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 3 & 1 \end{bmatrix}$$

⇨ excessive internal force *may* be generated

Application to tumbling operation by position-controlled manipulator





boundary that defines whether excessive internal force may be generated or not

5. Conclusion

We proposed two algorithms:

- Method to calculate object-stability measure
- Method to judge the possibility of excessive internal force

with numerical examples

Future Work

Application to the planning problem of graspless manipulation