Caging-Based Grasping by a Robot Hand with Rigid and Soft Parts

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***Abstract*— Caging is a method to make an object inescapable from a closed region by rigid bodies. Position-controlled robot hands can capture an object and manipulate it via caging without force sensing or force control. However, the object in caging is freely movable in the closed region, which may not be allowed in some applications. In such cases, grasping is required. In this paper, we propose a new simple approach to grasping by position-controlled robot hands with the advantage of caging: caging-based grasping by a robot hand with rigid and soft parts. In caging-based grasping, we cage an object with the rigid parts of the hand, and construct a complete grasp with the soft parts. We formulate the caging-based grasping, and derive concrete conditions for caging-based grasping in planar and spatial cases, and show some experimental results.**

I. Introduction

Constraining an object geometrically in a closed region by rigid bodies (Fig. 1, 2) is termed "caging" [1]. Caging can be performed by position-controlled robots based on geometrical information, and therefore it is easily executed by today's robots. It can be used as a substitute for or a complement to conventional grasping in robotic manipulation [2]–[15].

The object in caging is freely movable in the closed region unless the region is a single point (i.e., form closure). Some issues due to this nature of caging, such as difficulty in accurate positioning and possible collision between the object and the robot, may be unacceptable in some applications. In such cases, grasping is necessary.

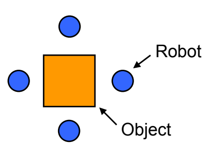
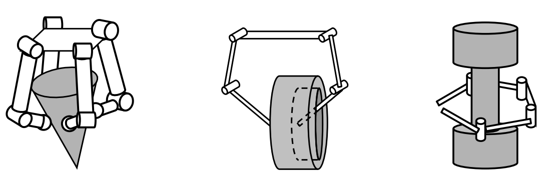


Fig. 1: 2D Caging



(a) Envelope-type caging (b) Ring-type caging (c) Waste-type caging

Fig. 2: 3D multifingered caging

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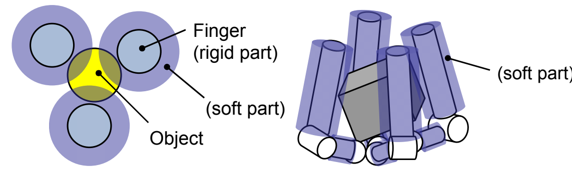


(a) A planar example (b) A spatial example

Fig. 3: Caging-based grasping

In this paper, we study a new simple approach to robotic grasping with the advantage of robotic caging. We propose "caging-based grasping" by a robot hand with rigid and soft parts (Fig. 3). In caging-based grasping, an object is caged by the rigid parts of the hand, and a complete grasp is achieved by the soft parts. Caging-based grasping enables position-controlled robot hands not only to cage but also to grasp objects without force sensing or force control.

There are many previous studies on grasping by position-controlled compliant robot hands (for example, [16] and [17]). However, these studies require stability analysis of grasps based on mechanical information such as structural and servo compliance. On the other hand, caging-based grasping can be achieved only by geometrical information such as robot configuration, object configuration and shape, and does not require explicit mechanical analysis of grasping that depends on contact friction and elasticity.

II. Caging-Based Grasping

A. Definition

Let us consider grasping an object by a robot hand with rigid and soft parts: the rigid parts like "bones" are covered with the soft parts like "flesh." When both of the following conditions hold, we call the situation "caging-based grasping":

1) Rigid-part caging condition: The object is caged in a closed region formed by the rigid parts of the robot hand.
2) Soft-part deformation condition: Assuming that the soft parts of the robot hand are rigid, the closed region for caging becomes empty.

From the former condition, the object is inescapable from a "cage" constructed by the rigid parts of the hand. Additionally, from the latter condition, the soft parts cannot keep their original shape and therefore deform as a reaction to the object. Consequently, the object is grasped by reaction forces of the deformed soft parts. Note that both of the conditions can be tested geometrically and explicit mechanical analysis is not necessary.

B. Formulation

Here we formulate caging-based grasping. Let us define the following symbols:

- n: The number of robot bodies.
- $\mathcal{C}$: The configuration space of the object.
- $\mathcal{A}_{\mathrm{obj}}(\boldsymbol{q})$: The object in the workspace when its configuration is $\boldsymbol{q} \in \mathcal{C}$.
- $\mathcal{A}_i$: The rigid part of the i-th robot body in the workspace ($i = 1, \ldots, n$).
- $\mathcal{A}'_i$: The rigid and soft parts of the i-th robot body ($i = 1, \ldots, n$) without deformation of the soft part.
- $\boldsymbol{q}_{\mathrm{obj}}$: The configuration of the object.

By definition, $\mathcal{A}_i \subset \mathcal{A}'_i$.

The free configuration space of the object in which the object is free from interference with the rigid robot bodies $\mathcal{A}_1, \ldots, \mathcal{A}_n$ can be written as follows:

$$\mathcal{C}_{\mathrm{free}} := \left\{ \boldsymbol{q} \in \mathcal{C} \,\middle|\, \mathcal{A}_{\mathrm{obj}}(\boldsymbol{q}) \cap \left(\bigcup_{i=1}^{n} \mathcal{A}_i \right) = \emptyset \right\}. \tag{1}$$

Similarly, the free configuration space of the object in which the object is free from interference with the robot bodies $\mathcal{A}'_1, \ldots, \mathcal{A}'_n$ can be written as follows:

$$\mathcal{C}'_{\mathrm{free}} := \left\{ \boldsymbol{q} \in \mathcal{C} \,\middle|\, \mathcal{A}_{\mathrm{obj}}(\boldsymbol{q}) \cap \left(\bigcup_{i=1}^{n} \mathcal{A}'_i \right) = \emptyset \right\}. \tag{2}$$

$\mathcal{C}'_{\mathrm{free}}$ is the free configuration space when assuming soft parts of the robots rigid. Naturally we obtain $\mathcal{C}'_{\mathrm{free}} \subseteq \mathcal{C}_{\mathrm{free}}$.

When the object is caged by the rigid parts of the object, $\mathcal{C}_{\mathrm{free}}$ can be separated into two parts: an inescapable configuration space (ICS) [2] and an escapable configuration space (ECS). If the former space does not exist, the object cannot be caged. In the latter space, the object can escape in the distance. Here we define the former space $\mathcal{C}_{\mathrm{free\_ICS}}$ and the latter space $\mathcal{C}_{\mathrm{free\_ECS}}$ as follows:

$$\mathcal{C}_{\mathrm{free\_ICS}} := \mathcal{C}_{\mathrm{free}} \backslash \mathcal{C}_{\mathrm{free\_ECS}} \tag{3}$$

$$\mathcal{C}_{\mathrm{free\_ECS}} := \bigcup_{\boldsymbol{q} \in \mathcal{Q}_{\mathrm{dist}}} \mathcal{C}_{\mathrm{free\_max}}(\boldsymbol{q}), \tag{4}$$

where $\mathcal{Q}_{\mathrm{dist}} (\subset \mathcal{C})$ is a set of object configurations that can be regarded as "distant"; $\mathcal{C}_{\mathrm{free\_max}}(\boldsymbol{q})$ is the maximal connected subset of $\mathcal{C}_{\mathrm{free}}$ that includes $\boldsymbol{q}$. Now the rigid-part caging condition can be written as follows:

$$\mathcal{C}_{\mathrm{free\_ICS}} \neq \emptyset \tag{5}$$

$$\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{free\_ICS}}. \tag{6}$$

When only the rigid-part caging condition ((5) and (6)) holds, the object can freely move in $\mathcal{C}_{\mathrm{free\_ICS}}$. On the other hand, if $\mathcal{C}_{\mathrm{free\_ICS}}$ is occupied by the soft parts of the robot hand, the object cannot exist without the deformation of the soft parts. In this case, the object is grasped by the soft parts and localized in $\mathcal{C}_{\mathrm{free\_ICS}}$. The soft-part deformation condition can be written as follows:

$$\mathcal{C}'_{\mathrm{free\_ICS}} = \emptyset, \tag{7}$$

where

$$\mathcal{C}'_{\mathrm{free\_ICS}} := \mathcal{C}'_{\mathrm{free}} \backslash \mathcal{C}'_{\mathrm{free\_ECS}} \tag{8}$$

$$\mathcal{C}'_{\mathrm{free\_ECS}} := \bigcup_{\boldsymbol{q} \in \mathcal{Q}_{\mathrm{dist}}} \mathcal{C}'_{\mathrm{free\_max}}(\boldsymbol{q}), \tag{9}$$

where $\mathcal{C}'_{\mathrm{free\_max}}(\boldsymbol{q})$ is the maximal connected subset of $\mathcal{C}'_{\mathrm{free}}$ that includes $\boldsymbol{q}$.

Thus the caging-based grasping can be formulated as the combination of (5), (6) and (7) as follows:

$$\begin{cases} \mathcal{C}_{\mathrm{free\_ICS}} \neq \emptyset, \\ \boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{free\_ICS}}, \\ \mathcal{C}'_{\mathrm{free\_ICS}} = \emptyset. \end{cases} \tag{10}$$

C. Another Formulation

The rigid-part caging condition can be written in another form. Caging is achieved when all the following conditions hold [14]:

- *Closed region formation.* The robot forms a closed region through which the object cannot pass:
$$\exists \mathcal{C}_{\mathrm{closed}} \text{ such that } \mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}_{\mathrm{free\_ECS}} = \emptyset. \tag{11}$$
- *Object inside.* The object is inside the closed region:
$$\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{closed}}. \tag{12}$$
- *No interference.* The object does not geometrically interfere with the robot bodies:
$$\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{free}}. \tag{13}$$

Note that only from (11) we cannot judge whether the object can exist in the closed region (that is, $\mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}_{\mathrm{free}} \neq \emptyset$) or not. The additional (12) and (13) guarantee (5) and (6), because $\mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}_{\mathrm{free}} \subseteq \mathcal{C}_{\mathrm{free\_ICS}}$. Equations (11)–(13) are the rigid-part caging condition in another form.

Let us assume the soft parts of the hand as rigid to reformulate the soft-part deformation condition. Caging should not hold for the enlarged rigid hand. However, (11) and (12) automatically hold for the same $\mathcal{C}_{\mathrm{closed}}$. Thus (13) must not hold.

A naive idea leads to just the negation of (13), $\boldsymbol{q}_{\mathrm{obj}} \notin \mathcal{C}'_{\mathrm{free}}$. However, this is not enough because there may exist different interference-free configurations in the cage, to which the object can move. Thus the object must interfere with the enlarged rigid hand in any configurations in $\mathcal{C}_{\mathrm{closed}}$. That is,

$$\mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}'_{\mathrm{free}} = \emptyset. \tag{14}$$

This is another form of the soft-part deformation condition.

Thus the caging-based grasping can be reformulated as the combination of the rigid-part caging condition (11)–(13) and the soft-part deformation condition (14) as follows:

$$\begin{cases} \exists \mathcal{C}_{\mathrm{closed}} \text{ such that } \mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}_{\mathrm{free\_ECS}} = \emptyset, \\ \boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{closed}},\ \boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{free}},\ \mathcal{C}_{\mathrm{closed}} \cap \mathcal{C}'_{\mathrm{free}} = \emptyset. \end{cases} \tag{15}$$

This formulation does not use explicit representation of the ICS ($\mathcal{C}_{\mathrm{free\_ICS}}$) and therefore it is less straightforward than the formulation in Section II-B. Due to this, however, we can derive concrete conditions of caging-based grasping more easily by defining $\mathcal{C}_{\mathrm{closed}}$ appropriately in many cases.

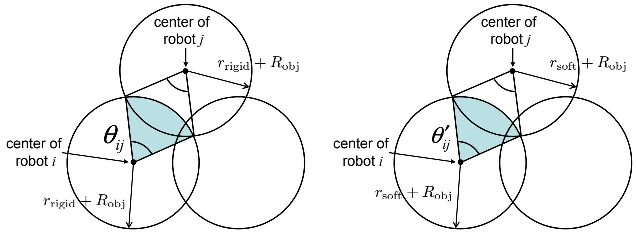
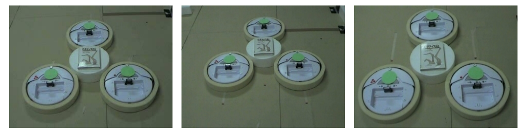


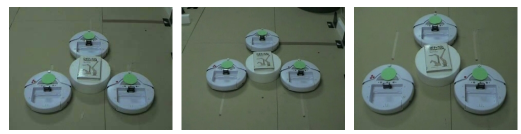
Fig. 4: Definition of θ_{ij} and θ'_{ij}

Fig. 5: Robot with soft parts



(a) (b) (c)

Fig. 6: Planar caging-based grasping: go and return



(a) (b) (c)

Fig. 7: Planar caging: go and return

III. 2D Caging-Based Grasping

As the simplest example, we consider grasping a circular object by three congruent circular robots (Fig. 3a). The robots can be regarded as "fingertips" of a robot hand. We make the following assumptions:

- The object is a rigid circle of radius R_{obj}.
- The robot is composed of a circular rigid part of radius r_{rigid} and a concentric circular soft part of radius r_{soft} ($r_{\text{rigid}} < r_{\text{soft}}$).

We do not care the orientation of the object and the robots because they are circular.

The object can be regarded as a point instead of a circle, if we grow robots by R_{obj}; that is, we consider caging-based grasping of a point object by three grown robots, which have circular rigid parts of radius $r_{\text{rigid}} + R_{\text{obj}}$ and concentric circular soft parts of radius $r_{\text{soft}} + R_{\text{obj}}$.

A. Derivation of Condition for Caging-Based Grasping

Let us derive the rigid-part caging condition for this case. The distance between the robots must be sufficiently small to prevent escaping of the object, as far as the rigid parts do not penetrate each other:

$$2r_{\text{rigid}} \le d_{ij} < 2(R_{\text{obj}} + r_{\text{rigid}}) \quad (i \neq j), \tag{16}$$

where d_{ij} is the distance between the centers of the i-th and j-th robots. Moreover, the following must hold so that $\mathcal{C}_{\text{free\_ICS}}$ is not empty:

$$\theta_{12} + \theta_{23} + \theta_{31} \le \pi, \tag{17}$$

where θ_{ij} is a central angle of a circular sector as shown in the left of Fig. 4 and given by:

$$\theta_{ij} = 2\cos^{-1}\frac{d_{ij}}{2(R_{\text{obj}} + r_{\text{rigid}})}. \tag{18}$$

The derivation of (17) can be found in Appendix.

Thus the rigid-part caging condition (5) is reduced to (16) and (17). In addition to this, the rigid-part caging condition requires (6), but testing it is trivial in experiments with actual robots.

Next, we derive the soft-part deformation condition (7) for this case. The closed region for caging by the rigid parts must be occupied by the soft parts. Assuming the soft parts as rigid, we can write this condition as the negation of (17) as follows (see also Appendix):

$$\theta'_{12} + \theta'_{23} + \theta'_{31} > \pi, \tag{19}$$

where θ'_{ij} is a central angle of a circular sector as shown in the right of Fig. 4 and given by:

$$\theta'_{ij} = 2\cos^{-1}\frac{d_{ij}}{2(R_{\text{obj}} + r_{\text{soft}})}. \tag{20}$$

Finally the condition for caging-based grasping (except for (6)) can be summarized from (16), (17) and (19) as follows:

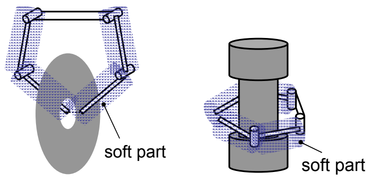
$$\begin{cases} 2r_{\text{rigid}} \le d_{ij} < 2(R_{\text{obj}} + r_{\text{rigid}}) \quad (i \neq j), \\ \theta_{12} + \theta_{23} + \theta_{31} \le \pi, \ \theta'_{12} + \theta'_{23} + \theta'_{31} > \pi. \end{cases} \tag{21}$$

B. Experiment

For validation of the feasibility of caging-based grasping, we conducted experiments with three mobile robots: iRobot Create (Fig. 5), which is almost circular of 168 [mm](= r_{rigid}) radius. We added circular urethane foam of 30 [mm] thickness around each of the robots ($r_{\text{soft}} = 168 + 30 = 198$ [mm]). The object was a disk of styrene foam of 150 [mm](= R_{obj}) radius used as a pallet, on which a book was placed as a load.

In the experiment shown in Fig. 6, the robots and the object were located so that $d_{ij} = 600$ [mm]. In this case, caging-based grasping was possible because (21) was satisfied. The robots translated forward and backward keeping their formation to manipulate the object, and the relative position among the object and the robots was almost maintained throughout the manipulation.

For comparison, a caging experiment by the robots without urethane foam was carried out. The robots and the object were located at the same initial positions, and translated in the same way. The object was also manipulated successfully, but the relative position among the object and the robots was not maintained when the robots returned to their initial positions (Fig. 7).



(a) Ring type (b) Waist type

Fig. 8: Typical types of caging-based grasping

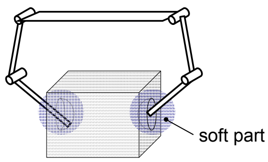


Fig. 9: Ring-type caging-based grasping using concavities

IV. 3D Caging-Based Grasping

A. Typical Types of 3D Caging-Based Grasping

The authors' group studied 3D multifingered caging and classified it into three typical categories [14]: "envelope-type caging" (Fig. 2a), "ring-type caging" (Fig. 2b) and "waist-type caging" (Fig. 2c). Similarly in 3D caging-based grasping, we can define "envelope-type caging-based grasping" (Fig. 3b), "ring-type ..." (Fig. 8a) and "waist-type ..." (Fig. 8b). We can use an appropriate type of caging-based grasping for applications.

Caging can be realized by adding some local geometric features to an object. For example, when we add a small "knob" on an object, the object can be caged by envelope-type caging of the knob. Similarly, caging-based grasping can be achieved using local geometric features. Fig. 9 is an example. Two additional hollows on the object enable ring-type caging-based grasping; the object is caged by the rigid parts at the fingertips placed in the hollows, and grasped by the soft parts at the fingertips. In this case, caging-based grasping of the object is achieved using only the local hollows and the shape of the object except for the hollows matters little. This will be useful, for example, to grasp occluded unknown objects.

B. Derivation of Sufficient Condition for Caging-Based Grasping

Here we show an example of 3D caging-based grasping. For simplicity, we consider a symmetric 2DOF three-fingered robot hand (Fig. 10) with the following features:

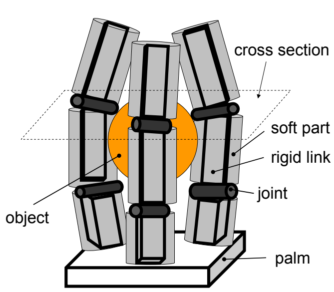
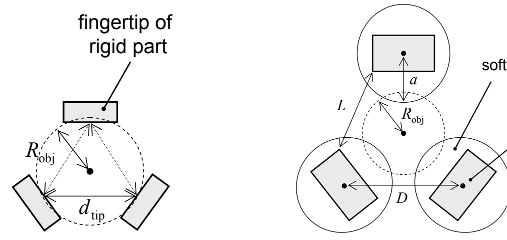


Fig. 10: A symmetric three-fingered hand



(a) at the fingertips (b) at the cross section with the maximal gap

Fig. 11: Schematic views of the robot hand

- The hand consists of a flat palm and three fingers.
- All the fingers are identical in there structure; each of them has three cuboid links as its rigid parts, covered with cylindrical soft parts and connected by two revolutionary joints.
- All the fingers are attached at the vertices of an equilateral triangle on the palm with circular symmetry.
- All the angles of the first and the second joints of the fingers are identical, respectively.

Also for simplicity, the object to be grasped is a sphere whose radius is $R_{\rm obj}$.

Generally it is difficult to derive the necessary and sufficient condition for caging-based grasping. However, as in the case of 3D multifingered caging [9] [14], a sufficient condition can be derived with relative ease.

First, let us derive a sufficient condition for rigid-part caging. The following is sufficient to prevent the sphere from escaping through the gap between the fingertips:

$$d_{\rm tip} < \sqrt{3} R_{\rm obj}, \tag{22}$$

where $d_{\rm tip}$ is the distance between the rigid parts at the fingertips as shown in Fig. 11a.

Fig. 11b shows the cross section of the hand, which is parallel to the palm and has the maximal gap between the rigid parts of the fingers. A sufficient condition to prevent the sphere from escaping through the side gap between two of the fingers can be written as follows:

$$L < 2R_{\rm obj}, \tag{23}$$

where L is the gap width depicted in Fig. 11b. Thus a sufficient condition of the rigid-part caging condition (11) can be reduced to (22) and (23). In addition to this, the rigid-part caging condition requires (12) and (13), but testing them is trivial in experiments with actual robots.

Next, let us derive a sufficient condition for soft-part deformation. If the soft parts are larger than a threshold at the cross section of the hand shown in Fig. 11b, the sphere cannot exist in the "cage" without deformation of the soft parts. This can be written as follows:

$$\sqrt{3}(a + R_{\rm obj}) > D, \tag{24}$$

where a is the major radius of the ellipse that is the cross section of the cylindrical soft part, and D is the distance between the centers of the ellipses, as shown in Fig. 11b.

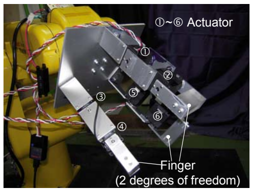


Fig. 12: Rigid parts of the hand

Fig. 13: The hand with soft parts

Fig. 14: 3D caging-based grasping

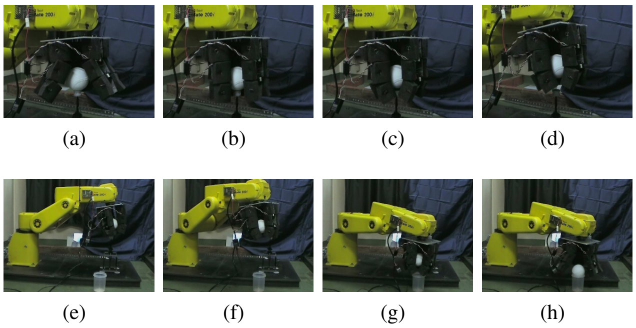


Fig. 15: Pick-and-place experiment

Thus a sufficient condition for caging-based grasping (except for (12) and (13)) can be summarized from (22), (23) and (24) as follows:

$$R_{\mathrm{obj}} > \max\left(\frac{d_{\mathrm{tip}}}{\sqrt{3}}, \frac{L}{2}, \frac{D}{\sqrt{3}} - a\right). \tag{25}$$

C. Experiment

For experimental validation of the feasibility of 3D caging-based grasping, we fabricated a three-fingered robot hand as shown in Fig. 12. Each finger has two RC servo motors (Futaba RS405CB). It was attached to an industrial manipulator, Fanuc LR-Mate 200iA.

For caging-based grasping, urethane foam was attached to each of the finger links of the hand (Fig. 13) to form a semicylindrical soft part.

Fig. 14 shows an example of 3D caging-based grasping of a softball of radius 42.5 [mm] ($= R_{\mathrm{obj}}$) by the hand. In this case, all the joints of the robot hand were position-controlled so that $d_{\mathrm{tip}} = 68$ [mm], $a = 45$ [mm], $L = 77$ [mm] and $D = 126$ [mm]; caging-based grasping was possible because the sufficient condition (25) held.

A pick-and-place experiment was also performed as shown in Fig. 15. The softball was picked up by caging-based grasping, carried to a destination, and placed successfully.

V. Discussion

We derived concrete conditions for caging-based grasping in two simple cases. Based on the derived conditions, caging-based grasping was successfully demonstrated by mobile robots and a multifingered robot hand.

Generally it is very difficult to derive a necessary and sufficient condition for caging-based grasping. However, as shown in the previous section, the derivation of a sufficient condition for caging-based grasping of a specific object by a specific hand is not very difficult. Moreover, the derived concrete conditions in Section III and IV can be used as sufficient conditions for caging-based grasping of objects in different shapes (e.g., see Fig. 16), by considering circles or spheres inscribed in the objects.

Fig. 16: Caging-based grasping of a rectangle

The "quality" of grasping depends on the softness of the soft parts. The selection of the softness is not sensitive, but should be appropriate. When the soft parts are not very soft, we should regard the rigid parts larger than their original shape (for example, r_{rigid} in Section III should be increased).

Note that caging-based grasping does not guarantee the immobilization of the object; the pose of the grasped object can be perturbed by external disturbances to some extent. This is also the case with objects in conventional compliant grasps. In fact, in caging-based grasping, it is guaranteed that the perturbation of the object is within the "cage."

VI. Conclusion

In this paper, we proposed a new simple approach to robotic grasping by position-controlled robot hands: caging-based grasping. It enables robot hands to grasp various objects without force sensing, force control or mechanical analysis of grasping. We formulated the caging-based grasping and derived concrete conditions for typical 2D and 3D cases. Experimental validation of caging-based grasping was also performed for 2D (by mobile robots) and 3D (by a multifingered hand) cases.

This paper is the first step of studies on caging-based grasping. There are many issues to be addressed, which include how to grasp and release objects, how to select appropriate softness for soft parts, and how to derive concrete conditions for caging-based grasping of various objects by various robot hands.

Acknowledgment

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APPENDIX
DERIVATION OF (17)

Let us grow the rigid parts of the robots by R_{obj} so that we can regard the object as a point. When the condition on the inter-robot distance (16) holds, there are four patterns of mutual robot positions as shown in Fig. 17. The circles in the figure, whose radius is $r_{\mathrm{rigid}} + R_{\mathrm{obj}}$, correspond to the grown rigid parts.

Because the sum of the interior angles of a hexagon is 4π [rad], in the case of Fig. 17a, where $\mathcal{C}_{\mathrm{free\_ICS}} = \emptyset$, the following holds:

$$\begin{aligned} &2(\theta_{12}+\theta_{23}+\theta_{31}) + (2\pi - 2\theta_{12})/2 \\ &\quad + (2\pi - 2\theta_{23})/2 + (2\pi - 2\theta_{31})/2 > 4\pi. \\ &\qquad \therefore \theta_{12}+\theta_{23}+\theta_{31} > \pi. \end{aligned} \tag{26}$$

In the case of Fig. 17b, where $\mathcal{C}_{\mathrm{free\_ICS}}$ is a point (i.e., not empty), considering the interior angles of the hexagon in the figure leads to:

$$\theta_{12}+\theta_{23}+\theta_{31} = \pi. \tag{27}$$

Similarly, in the case of Fig. 17c, where $\mathcal{C}_{\mathrm{free\_ICS}} \neq \emptyset$,

$$\theta_{12}+\theta_{23}+\theta_{31} < \pi. \tag{28}$$

In the case of Fig. 17d, $\mathcal{C}_{\mathrm{free\_ICS}} = \emptyset$. Instead of considering hexagons, we adopt a different approach for this case. Without loss of generality, we assign the number of circles as shown in the figure.

Let us move circle 3 in the direction perpendicular to the line between the centers of circle 1 and 2, so that the intersection of the three circles becomes a point. We call the moved one circle 3' (see Fig. 17d). Let us denote the distance between circle 3' and 1, and that between circle 2 and 3', by $d_{3'1}$ and $d_{23'}$, respectively. Obviously the following holds:

$$d_{23} < d_{23'}, \quad d_{31} < d_{3'1}. \tag{29}$$

From (18), θ_{ij} increases monotonically when d_{ij} decreases. Therefore, from (29),

$$\theta_{23} > \theta_{23'}, \quad \theta_{31} > \theta_{3'1}. \tag{30}$$

Because the intersection of the three circles becomes a point, we obtain the following from (27):

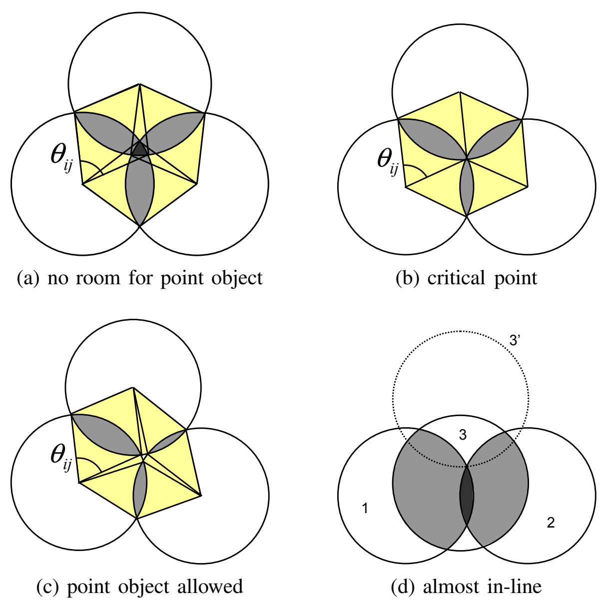
$$\theta_{12}+\theta_{23'}+\theta_{3'1} = \pi. \tag{31}$$

Thus from (31) and (30), (26) holds also in the case of Fig. 17d.

The summary of the above is as follows:

$$\begin{cases} \theta_{12}+\theta_{23}+\theta_{31} \leq \pi & \text{when } \mathcal{C}_{\mathrm{free\_ICS}} \neq \emptyset, \\ \theta_{12}+\theta_{23}+\theta_{31} > \pi & \text{when } \mathcal{C}_{\mathrm{free\_ICS}} = \emptyset. \end{cases} \tag{32}$$

Thus (17) holds when $\mathcal{C}_{\mathrm{free\_ICS}}$ is not empty.



(a) no room for point object (b) critical point

(c) point object allowed (d) almost in-line

Fig. 17: Patterns of relative robot positions