## A Quantitative Stability Measure for Graspless Manipulation

O Yusuke MAEDA and Tamio ARAI
(The University of Tokyo)

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## 1. Introduction

## Graspless (Nonprehensile) Manipulation

to Manipulate Objects without Grasping [Aiyama 93]


- No need to support all the weight of objects
- Complement to conventional pick-and-place


## Disadvantage of Graspless Manipulation

## Less Stability than Pick-and-Place

- Not Form- nor Force-Closure


Evaluation of manipulation stability is important

## Stability of Graspless Manipulation



## Related Works

[Mason and Lynch 93]
..."Quasi-Static Closure" "Dynamic Closure"
[Trinkle 95]... "First-Order Stability"
[Maeda et al. 96]...Quantitative Stability Measure for
Manipulation without Sliding Contacts
[Yu and Yoshikawa 97]..."Contact Maintainability"
[Kijimoto et al. 99]... Quantitative Stability Measure for Graspless Manipulation with Little Physical Basis

## Objective

## Quantitative Stability Measure for Graspless Manipulation

- Consideration to gravity and friction
- Applicable to not only pushing but also other graspless operations


## 2. Model of Contact Forces

## Assumptions

- Quasi-Static manipulation of a polyhedral object
- Under gravity and Coulomb friction
- Friction coefficient is uniform on each contact surface
- Static and kinetic friction coefficients are equal
- Each friction cone can be approximated as a polyhedral convex cone



## Set of Applicable Contact Forces

Set of Generalized Forces Applicable to Object through Point Contacts

Represented as Union of Polyhedral Convex Cones
[Yu and Yoshikawa 97, 01]

Approximate all the contacts with point contacts

## Friction on Surface Contact

Representative Points


Stationary or in Translation

in Rotation

## Surface Contact in Rotation

## COR = Center Of Rotation

- Case 1: Instantaneous COR is outside the contact surface

Instantaneous

- Case 2: Instantaneous COR is on the contact surface



## Case 1: COR is outside the contact surface



Contact forces on each half-line have the same direction vector

## Approximation by <br> Finite Representative Points

## Case 2: COR is on the contact surface



Contact forces on each half-line have the same direction vector

## 3. New Stability Measure

## Two Types of Stability of Graspless Manipulation

1. Ability of manipulated objects to resist disturbing force without changing their motion
2. Ability of manipulated objects to resume their original motion after a perturbation by disturbing force

## Our Stability Measure

Magnitude of disturbing (generalized) force that the object can resist without changing its motion

$$
z=\min _{\left\|\hat{Q}_{\text {dist }}\right\|_{M}=1} \max _{Q_{\text {known }}+Q=-t \hat{Q}_{\text {dist }},}\left\|\boldsymbol{Q}_{\text {known }}+\boldsymbol{Q}\right\|_{M}
$$

$\boldsymbol{Q}$ : Resultant Contact Force
$\boldsymbol{Q}_{\text {known }}$ : Known External Force (gravity, etc.)
$\hat{\boldsymbol{Q}}_{\text {dist }}$ : Direction Vector of (Unknown) Disturbing Force

$$
\|\boldsymbol{Q}\|_{\boldsymbol{M}}=\sqrt{\boldsymbol{Q}^{T} \boldsymbol{M}^{-1} \boldsymbol{Q}}
$$

M : Inertia Matrix of Object

## Discussion about Stability Measure

Stability Value $z$ :
Magnitude of Resistible Disturbance in the "Weakest" Direction
$z>0 \quad$... Disturbance smaller than $z$ cannot perturb object motion
$\square$ Stable Manipulation
$z=0 \quad \ldots$ Infinitesimal disturbance can perturb object motion
$\square$ Unstable Manipulation

## Calculation of the Stability Value by Linear Programming


(Numerical Examples: 76 vertices)

## 4. Numerical Examples

## Graspless Manipulation of a Cuboid

Manipulation by Two Position-Controlled Robot Fingers


Size of Object: $2 \times 2 \times 1$
Mass of Object: 1
Friction coefficient: 0.2

## Translation by Pushing



## Rotation by Pushing



Stability $=4.2$
$\left\{\begin{array}{l}4.1 \\ 0.7\end{array}\right.$
Stable

Calculation Time:
1.3 CPU Seconds
(Pentium4-1.5GHz)

## 6. Conclusion

## Summary

- A quantitative stability measure for graspless manipulation
- Calculation method of the measure by linear programming


## Future Work

- Application to Planning of Graspless Manipulation


